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On the theory of diffracted waves by transmission echelon grating in optical and quasi-optical millimetre wave region

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Abstract : A thorough mathematical analysis has been made with regard to the intensity of the diffracted light by transmission echelon grating. It appears that the grating could be used satisfactorily even in the millimetre wave region if proper dielectric is chosen for minimum absorption. A synthetic material, Mylar, possesses such properties. The results suggest that the optimum intensity response depends upon the geometrical parameters of the grating and also the operating wavelength.

Keywords : Diffraction, optical and millimetre waves, transmission echelon grating

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1. Introduction

Michelson [1,2] first devised echelon transmission grating by arranging a number of glass plates in the form of steps as shown in Figure 1. If a well collimated beam is allowed to pass through the echelon, each step behaves like an aperture which produces a single slit pattern centred on the direction of normal transmission. Although it has greater power of concentrating light to one particular order than the plane transmission grating, it became almost obsolete because the Fabry Perot etalon can give better resolution at a much smaller cost [3]. With the rapid development in the microwave spectroscopy [4,5], it appears that the transmission echelon can serve well with the millimetre waves if we choose a proper dielectric instead of glass so as to have minimum absorption for such waves. One such substance is a synthetic material, Mylar, which is transparent to both optical and millimetre waves with negligible absorption. If we use millimetre waves obtained from a free electron beam laser or

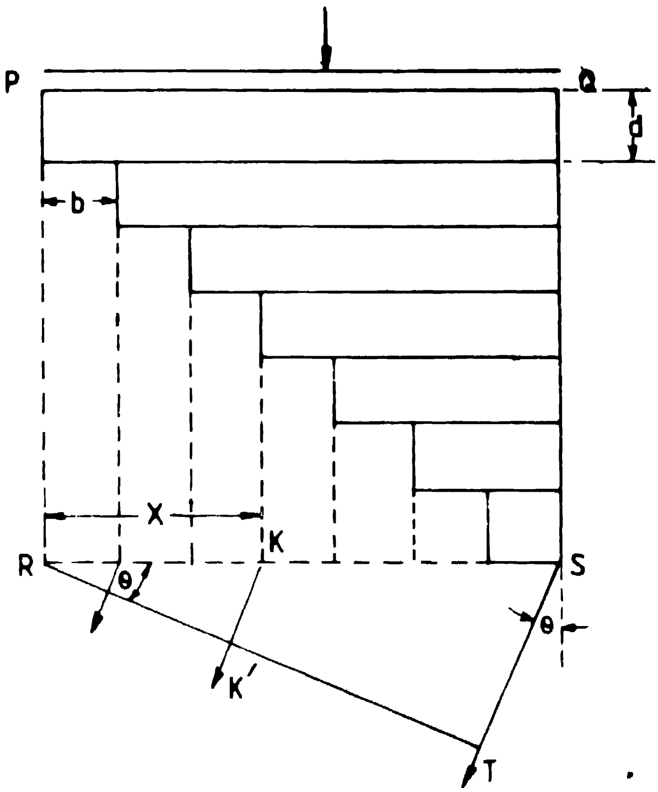

Figure 1. Ray diagram of the echelon transmission grating is shown with principal geometrical parameters

semi-conductor laser, the diffracted intensity of the transmitted beam is expected to supersede all the conventional diffraction gratings. So, the importance of transmission echelon, the pioneer work of which was done by Michelson, seems to have come again into the forefront. The application of laser may also prove to be of better performance with such echelons in the optical regions as well, because of their self-focussing phenomenon [6–8]. It will not be out of place to mention that several optical principles like that of zone plates also found to be useful in the construction of microwave zone-plate antenna [9–16] and other microwave devices [17–20]. With the advent of lasers and holographic techniques [21–24], it is now possible to have gratings with much higher resolution and luminosity. The holographic gratings are free from aberrations [24,25]. In this context, the transmission echelon grating illuminated with laser or maser in conjunction with holographic technique may play a promising role in the near future. Since the standard literatures [26–28] do not give much information regarding the intensity distribution of the transmitted light, it is worth while to analyse this aspect more rigorously for such echelons and consider its practical utility for millimetre waves in addition to the optical waves

2. Theoretical details

Let a parallel beam of light be incident on the upper surface of the echelon (Figure 1)

Let the equation of the incident wave front PQ be

$$y = \sin \frac{2\pi t}{T} \tag{1}$$

assuming the amplitude of vibration to be unity.

The path difference between PQ and RS of the portion of the wave front, after transmission through the p-th plate, is given by

$$l_1 = Nd + p(\mu - 1)\, d \tag{2}$$

where N is the number of steps, d is the depth and μ is the refractive index of the echelon plates

The equation of vibration at the point K in the plane RS under the p-th step is given by,

$$y = \sin \frac{2\pi}{T}\left(t - \frac{l_1}{c}\right) \tag{3}$$

where c is the velocity of light.

Let the distance RK measured from the origin R be x If the angle of diffraction be θ, then the path difference l_2 between the incident and diffracted beam is given by,

$$l_2 = KK' = RK \sin\theta = x \sin\theta \tag{4}$$

Hence the equation of vibration at K' in the wave front RT can be expressed as,

$$\begin{aligned} y &= \sin 2\pi\left\{\frac{t}{T} - \frac{l_1 + l_2}{\lambda}\right\} \\ &= \sin 2\pi\left\{\frac{t}{T} - \frac{Nd}{\lambda} - \frac{p(\mu - 1)d}{\lambda} - \frac{x\sin\theta}{\lambda}\right\} \\ &= \sin(\alpha - p\beta - \gamma x) \end{aligned} \tag{5}$$

where

$$\left.\begin{aligned} \alpha &= 2\pi\left(\frac{t}{T} - \frac{Nd}{\lambda}\right) \\ \beta &= \frac{2\pi(\mu - 1)d}{\lambda} \\ \gamma &= \frac{2\pi \sin\theta}{\lambda} \end{aligned}\right\} \tag{6}$$

The disturbance at the focus coming from an element dx of the wave front at a distance x from R is

$$dS = \sin(\alpha - p\beta - \gamma x)\, dx$$

Substituting $\exp(-i(\alpha - p\beta - \gamma x))$ for $\sin(\alpha - p\beta - \gamma x)$

we have, $$dS = \exp(-i(\alpha - p\beta - \gamma x))\, dx \,. \tag{7}$$

The total disturbance S at the focus is obtained by putting $p = 1, 2, 3$ etc. in the successive integrals,

$$S = \exp(-i\alpha)\left[\int_0^b \exp(i(\beta + \gamma x)) + \int_b^{2b} \exp(i(2\beta + \gamma x)) + \cdots \right.$$

$$\left. \cdots + \int_{(N-1)b}^{Nb} \exp(i(N\beta + \gamma x))dx\right]$$

(where b is the breadth of each step)

$$= \frac{1}{i\gamma} \exp(-i(\alpha - \beta))(\exp(ib\gamma) - 1)\,[1 + \exp(i(\beta + \gamma b)) +$$

$$\exp(2i(\beta + \gamma b)) + \cdots + \exp((N-1)i(\beta + \gamma b))]$$

$$= \frac{\exp(-i(\alpha - \beta))(\exp(ib\gamma) - 1)}{i\gamma} \times \left[\frac{1 - \exp(iN(\beta + \gamma b))}{1 - \exp(i(\beta + \gamma b))}\right]. \tag{8}$$

Multiplying the complex amplitude by its complex conjugate, the intensity J which is the square of the amplitude is given by,

$$J = \left\{\frac{\exp(ib\gamma) - 1}{(i\gamma)}\right\}\left\{\frac{\exp(-ib\gamma) - 1}{(-i\gamma)}\right\}\left\{\frac{1 - \exp(iN(\beta + \gamma b))}{1 - \exp(i(\beta + \gamma b))}\right\}$$

$$\times \left\{\frac{1 - \exp(-iN(\beta + \gamma b))}{1 - \exp(-i(\beta + \gamma b))}\right\}$$

$$= \frac{\left(b \sin \dfrac{\gamma b}{2}\right)^2}{\left(\dfrac{\gamma b}{2}\right)^2} \; \frac{\left\{\sin \dfrac{N(\beta + \gamma b)}{2}\right\}^2}{\left\{\sin \dfrac{(\beta + \gamma b)}{2}\right\}^2}$$

$$= \left\{\frac{b \sin\left(\dfrac{\pi b \sin\theta}{\lambda}\right)}{\dfrac{\pi b \sin\theta}{\lambda}}\right\}^2 \times \left\{\frac{\sin \dfrac{\pi N\{(\mu - 1)d + b\sin\theta\}}{\lambda}}{\sin \dfrac{\pi\{(\mu - 1)d + b\sin\theta\}}{\lambda}}\right\}^2 . \tag{9}$$

The equation of vibration at the focus neglecting the imaginary quantity is given by,

$$S = \left\{ \frac{b \sin \frac{\pi b \sin\theta}{\lambda}}{\frac{\pi b \sin\theta}{\lambda}} \right\} \left\{ \frac{\sin \frac{\pi N\{(\mu - 1) d + \sin\theta\}}{\lambda}}{\sin \frac{\pi\{(\mu - 1) d + b \sin\theta\}}{\lambda}} \right\}$$

$$\times \sin 2\pi \left\{ \frac{t}{T} - \frac{d}{\lambda}(N + \mu - 1) \right\}. \tag{10}$$

It may be pointed out in this connection that the diffracted intensity can also be derived by using Fresnel's Fourier transform technique It was found that the intensity expression when calculated from this alternative approach, remains identical Hence we can conclude that the eq (9) for diffracted intensity derived from classical Fresnel–Huygen theory can be taken to be correct. The correctness becomes obvious when we attempt to calculate the dispersive and resolving power of the grating

The eq (9), for the intensity is of the same form as that for a grating for normal incidence Hence the principal maxima is given by.

$$(\mu - 1) d + b\theta = n\lambda \tag{11}$$

in the second factor of the intensity expression where $n = 0, 1, 2$. etc

Differentiating eq. (11) with respect to λ, we get

$$d\theta = \frac{n d\lambda - d(d\mu)}{b}$$

Since θ is small, $n \approx \frac{(\mu - 1) d}{\lambda}$ so that

$$\frac{d\theta}{d\lambda} = \frac{d}{b\lambda}\left[(\mu - 1) - \lambda . \frac{d\mu}{d\lambda}\right] = \frac{d}{b\lambda} D \tag{12}$$

where $$D = (\mu - 1) - \lambda \frac{d\mu}{d\lambda} \tag{13}$$

Hence, the dispersion $\frac{d\theta}{d\lambda}$ is proportional to the thickness d of the plates and is inversely proportional to the breadth b of the steps

The resolving power of such a grating can also be obtained by using Rayleigh criterion of resolution. The angular separation $d\theta$ between the principal maxima of λ and $\lambda + d\lambda$ is given by the eq. (12), so that

$$d\theta = \frac{d.D}{b.\lambda} . d\lambda \tag{14}$$

The transmission echelon grating containing N steps each of breadth b might be regarded as a plane transmission grating each of breadth b so that the angular separation $d\theta$ between the principal maximum of a given order and its first minimum of λ for normal incidence is obtained as

$$d\theta = \frac{\lambda}{Nb} \tag{15}$$

Hence from (14) and (15) we obtain,

$$\frac{\lambda}{d\lambda} = \frac{N\,d.D}{\lambda} = \frac{N.d.\left[(\mu - 1) - \lambda \dfrac{d\mu}{d\lambda}\right]}{\lambda} \tag{16}$$

Neglecting $b\theta$, we can write $(\mu - 1)\,d = n\lambda$. Neglecting also $\dfrac{d\mu}{d\lambda}$, an approximate expression for the resolving power can be obtained as,

$$\frac{\lambda}{d\lambda} = \frac{N\,d(\mu - 1)}{\lambda} = N.n \tag{17}$$

3. Results and discussion

The resolving power for the echelon transmission grating as given by eq. (16) is in complete agreement with the standard results [26]. This further substantiates the validity and correctness of our results obtained for the intensity of the echelon grating.

If we look into the eq. (17), the resolving power of the transmission echelon is found almost proportional to the number of steps N and the order number n, of the spectra. It is quite interesting to note that the result is similar to that of a plane transmission grating where N represents the total number of ruled lines and n, the order of the spectra.

From the eq. (16), it is quite evident that the resolving power is proportional to the depth and is inversely proportional to the wavelength. Its resolving power being dependent upon the value $\mu - 1 \simeq 0.5$, it is inferior to that of a reflecting echelon grating. Since the millimetre waves are much larger than the optical waves, and the technical difficulties limit the depth d, the resolving power of millimetre wave echelon transmission grating will not be as high as in the optical region. For example if $N = 30$, $d = 1$ cm, $\lambda = 0.1$ cm, the resolving power,

$$\frac{\lambda}{d\lambda} = \frac{30 \times 1 \times (1.5 - 1)}{0.1} = 150$$

This is about one fourth of that of the resolving power of reflection echelon grating with same step parameters. If d is made 1 cm, the resolving power increases to 1500 for a millimetre wave which is still much smaller than the resolving power of an optical echelon.

In the optical region, the order number of spectra through a transmission echelon is very high and as a consequence, there is much possibility of overlapping. This is perhaps one of the reason for its limited use. Now a days, it is mostly used in the study of Zeeman effect

In this respect however, a millimetre wave echelon appears to be superior because the chance of overlapping is extremely small in the region concerned, although its resolving power is not so high. In fact, the high resolution is not of primary importance in millimetre wave spectroscopy but the essential condition is to concentrate the diffracted electromagnetic energy as much as possible to a particular order. This has been well achieved in a millimetre wave transmission echelon

Let us now confine our attention to the intensity of the transmitted electromagnetic energy which will depend upon the values of N, b, d and λ. If we use Mylar, as the grating material, its absorption coefficient for millimetre waves being negligible, its intensity will not be much reduced. Taking $N = 30$, $b = 1$ cm, $d = 1$ cm, $\mu = 1.5$ and keeping θ within 5°, the plot of intensity J, *vs* wavelength λ is represented in Figure 2 for optical region and in Figure 3 for millimetre wave region respectively

It is evident from the Figure 2, that the maximum peak occurs at about 5 80 A.U for the optical region The Figure 3, however, shows the existence of several peaks in the

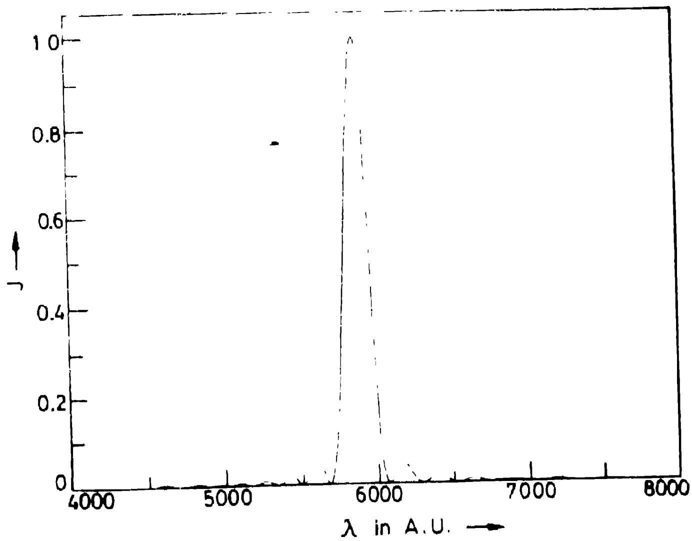

Figure 2. Intensity distribution as a function of the wavelength in the optical region The parameters are $N = 30$, $b = d = 1$ cm $\mu = 1.5$ $\theta = 5°$

millimetre wave region around 1 2 mm, 1 5 mm, 2 mm, 3 mm and 5 9 mm, the maximum peak being at 5.9 mm. Both these figures seem to suggest that the intensity response depends upon the geometrical parameters of the grating Hence, it can be concluded that a judicious selection of geometrical parameters of the echelon should be made in order to have optimum intensity response, which of course, will depend upon the operating wavelength as well

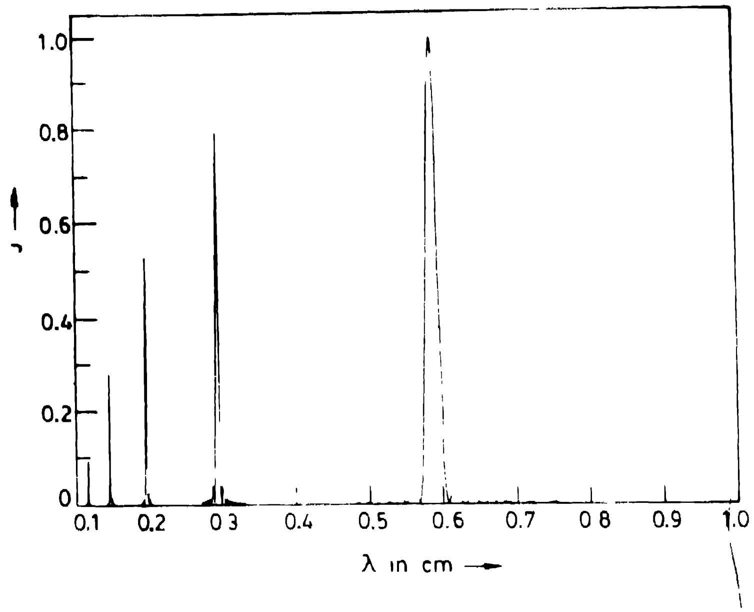

Figure 3. Intensity distribution as a function of the wavelength in the microwave region. The parameters are $N = 30$, $b = d = 1$ cm, $\mu = 1.5$, $\theta = 5°$

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